Benha University Faculty of Engineering – Shoubra Engineering and Management of Construction Sites Department Duration: 2 hours		Final Exam Course: Mathematics 3 Code: EMP 201 Date : December 23, 2017

The exam consists of one page No. of questions : 4 Answer **All** questions Total Mark: 40

Question 1

(a)Find the first derivatives of the function :

$f(x,y,z) = z\sin x + y\,e^{y} + \ln z$ — 3

(b)Find the envelope of the curves : $x\cos\alpha + y\sin\alpha = 2.$ — 3

(c)Determine the extrema of the function :

$\mathrm{f(x,y) = xy}$ subject to $\mathrm{g(x,y) = x^2 + 4y^2 - 32 = 0}$ — 4

Question 2

(a) Find $\nabla.\overline{\mathrm{U}}$ and $\nabla \mathrm{x}\, \overline{\mathrm{U}}$ where : $\overline{\mathrm{U}} = (\mathrm{x^2y})\mathrm{i} + (\mathrm{yz})\mathrm{j} + (\mathrm{xz} + \cos \mathrm{z})\mathrm{k}.$ — 4

(b) Find the integrals :

(i) $\iint_D \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$, D is bounded by $x^2 + y^2 = 4$, $x,\ y \geq 0$ — 4

(ii) $\int_{(0,0)}^{(1,2)} (x^2 + y)dx + (x - y^2)dy$ on the curve $y = 2x^3$ or $y = 2x$ — 4

Question 3

(a)Find u and v of the function : $f(z) = e^{2z}$ and show that they are harmonic. — 3

(b)If C is the circle | z – 2i | = 1, find the integral $\oint_C \frac{\cos 3z}{z^2-16}\,dz$ — 2

(c)If C is the circle | z | = 1, find the integral $\oint_C \frac{\ln(4+z)}{z}\,dz$ — 2

(d)If C is the circle | z – 1 | = 5, find the integral $\oint_C \frac{z^3}{(z+1)(z-2)}\,dz$ — 3

Question 4

(a)Find the integral $\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2+4}\,dx$ — 4

(b)Find the Fourier series of the function :

$f(x) = x + 1, \quad -1 \leq x \leq 1, \quad f(x+2) = f(x)$ — 4

Good Luck *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a) $f_x = z.\cos x, \quad f_y = y\,e^y + e^y, \quad f_z = \sin x + \frac{1}{z}$

--*3- Marks*

(b)Differentiate with respect to α, we get : $-x\sin\alpha + y\cos\alpha = 0$. Then $\tan\alpha = \frac{y}{x}$

The envelope is : $x\,\frac{x}{\sqrt{x^2+y^2}} + y\,\frac{y}{\sqrt{x^2+y^2}} = 2$ Or $x^2 + y^2 = 4$

--*3- Marks*

(c)From : $xy = \lambda(x^2 + 4y^2 - 32)$.

Then $y = \lambda(2x)$, $x = \lambda(8y)$. Then we get $\lambda = \frac{y}{2x} = \frac{x}{8y}$. Then $x^2 = 4y^2$

Substitute in g(x, y) : $4y^2 + 4y^2 = 32$. Then $y^2 = 4$. Then $y = 2, -2$ and $x = 4, -4$

Then, we get the points (4, 2), (4,– 2), (– 4, 2), (– 4, –2).

We see that f(4, 2) = f(– 4, –2) = 8 which is maximum.

and f(4, –2) = f(– 4, 2) = – 8 which is minimum.

--*4- Marks*

Answer of Question 2

(a) $\nabla.\overline{U} = 2xy + z + x - \sin z$.

$$\nabla x\overline{U} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2y & yz & xz + \cos z \end{vmatrix} = (0 - y)i - (z - 0)j + (0 - x^2)k$$

--*4- Marks*

(b) Put $x = r\cos\theta, y = r\sin\theta, dxdy = r\,dr\,d\theta$. Then

$$I = \int_0^{\frac{\pi}{2}}\int_0^2 \frac{r\cos\theta}{r}\, r\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\int_0^2 r\cos\theta\ dr\,d\theta = 2\int_0^{\frac{\pi}{2}}\cos\theta\ d\theta = 2$$

(ii) We see that $p_y = q_x$, from y = 2x and dy = 2 dx. Then

$$I = \int_0^1 (4x - 7x^2)\,dx = -\frac{1}{3}$$

--*8- Marks*

Answer of Question 3

(a) $f(z) = e^{2z} = e^{2x}\cos 2y + i\,e^{2x}\sin 2y$

Then $u = e^{2x}\cos 2y$ and $v = e^{2x}\sin 2y$

We see that $u_{xx} + u_{yy} = 0$ *and* $v_{xx} + v_{yy} = 0$. Then u *and* v are harmonic.

--*3- Marks*

(b) We see that the points 4, – 4 outside the circle | z – 2i | = 1.

Then $\oint_C \frac{\cos 3z}{z^2-16} dz = 0$

---*2- Marks*

(c) We see that the point 0 inside and – 4 outside the circle | z | = 1.

Then $\oint_C \frac{\ln(4+z)}{z} dz = 2\pi i \ln(0+4) = 2\pi i \ln 4$

---*2- Marks*

(d)Since the two points 2 , – 1 inside the circle | z – 1 | = 5. Then

$$\operatorname*{Res}_{z=-1} f(z) = \lim_{z\to -1} (z+1).\frac{z^3}{(z+1)(z-2)} = \frac{1}{3}$$

$$\operatorname*{Res}_{z=2} f(z) = \lim_{z\to 2} (z-2)\frac{z^3}{(z+1)(z-2)} = \frac{8}{3}$$

Then $I = 2\pi i \left(\frac{1}{3} + \frac{8}{3}\right) = 6\pi i$

---*3- Marks*

Answer of Question 4

(a) Since $\cos 2z = \operatorname{Re} e^{2iz}$. Then $f(z) = \frac{e^{2iz}}{z^2+4}$ has simple pole at the point 2i in upper half plane. Then

$$\operatorname*{Res}_{z=2i} f(z) = \lim_{z\to 2i} (z-2i)\frac{e^{2iz}}{(z+2i)(z-2i)} = \frac{e^{-4}}{4i}$$

Then $\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2+4} dx = \operatorname{Re}\left(2\pi i \frac{e^{-4}}{4i}\right) = \frac{\pi}{2e^4}$

---*4- Marks*

(b) $a_0 = \frac{1}{1}\int_{-1}^{1}(x+1)\, dx = 2$

$$a_n = \frac{1}{1}\int_{-1}^{1}(x+1)\cos n\pi x\, dx = \left[\frac{(x+1)\sin n\pi x}{n\pi} + \frac{\cos n\pi x}{(n\pi)^2}\right] = 0$$

$$b_n = \frac{1}{1}\int_{-1}^{1}(x+1)\sin n\pi x\, dx = \left[\frac{-(x+1)\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\right] = \frac{2}{n\pi}(-1)^{n+1}$$

Then $f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi} \sin n\pi x$

---*4- Marks*

Dr. Mohamed Eid